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## ideas with IMPACT



## Rhythms, Rhymes, and Multiplication Times



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## Goals and Objectives

Rhythms, Rhymes, and Multiplication Times have the ability to empower students to reach mastery of basic multiplication concepts and facts. Reading, Language Arts, Math, and Technology are integrated into a unit to aide teachers in providing a solid foundation for mastery of this critical concept and skill. Although students are introduced to multiplication by the end of second grade in most curriculums, a true understanding of the concept of multiplication and mastery with fluency of the facts is many times inadequate for future success in higher level mathematics courses. In this session teachers will learn strategies that can be used to introduce students to the concept of multiplication and to build fluency of the facts. They will engage in activities that incorporate rhythms, rhymes, movement and games as a means of learning about multiplication. Through these interactive activities that address a variety of learning styles, teachers will begin to understand how to help their students conquer the challenging multiplication tables of $6,7,8$, and 9 , which can easily become a "square of despair". As teachers implement the strategies to assist their students in learning about multiplication and moving toward mastery in fluency of the facts, they will begin to see improvement in the students' skills in the eight Mathematical Practices.

My experience has been that young children are excited to learn the multiplication facts. However, if they do not develop an understanding of the concept of multiplication and they do not develop fluency with the facts, math becomes a source of frustration because they need these foundational skills in order to move forward in solving more advanced mathematical problems involving multi-digit multiplication, fractions, ratios, division, and decimals. Conceptualization of
multiplication and fact fluency are prerequisite skills that, if not developed, will impede student's mathematical progress. For example if finding the solution to a math problem requires the recall of basic multiplication facts, and students lack understanding of the concept and/or the ability to quickly retrieve a basic fact, they will be more focused on the fact than the actual solving of the problem. They may be further disadvantaged whenever they are not be permitted to use a calculator to find the answer. Their choices are then limited to guessing or giving up, of which neither is acceptable. If they have been taught test-taking skills, they may guess correctly every now and then but without confidently knowing that their choice is correct. The end result will be the inability and/or lack of motivation to move forward.


## Mathematics Florida Standards (MAFS)

| CLUSTER | $\begin{array}{c}\text { STANDARD } \\ \text { CODE }\end{array}$ | STANDARD |
| :--- | :--- | :--- |
| $\begin{array}{l}\text { Work with equal groups of } \\ \text { objects to gain foundations } \\ \text { for multiplication. }\end{array}$ | MAFS.2.OA.3.3 | $\begin{array}{l}\text { Determine whether a group of objects (up to 20) has } \\ \text { an odd or even number of members, e.g., by pairing } \\ \text { objects or counting them by 2s; write an equation to } \\ \text { express an even number as a sum of two equal } \\ \text { addends. }\end{array}$ |
|  | MAFS.2.OA.3.4 | $\begin{array}{l}\text { Use addition to find the total number of objects } \\ \text { arranged in rectangular arrays with up to } 5 \text { columns; } \\ \text { write an equation to express the total as a sum of } \\ \text { equal addends. }\end{array}$ |
| Understand place value. | MAFS.2.NBT.1.3 | $\begin{array}{l}\text { Read and write numbers to 1000 using base-ten } \\ \text { numerals, number names, and expanded form. }\end{array}$ |
| $\begin{array}{l}\text { Represent and solve } \\ \text { problems involving } \\ \text { multiplication and division. }\end{array}$ | MAFS.3.OA.1.3 | $\begin{array}{l}\text { Use multiplication and division within 100 to solve } \\ \text { word problems in situations involving groups, arrays, } \\ \text { and measurement quantities, e.g., by using drawings } \\ \text { and equations with a symbol for the unknown number } \\ \text { to represent the problem. }\end{array}$ |
|  | MAFS.3.OA.1.4 | $\begin{array}{l}\text { Determine the unknown whole number in a } \\ \text { multiplication or division equation relating three } \\ \text { whole numbers. For example, determine the unknown } \\ \text { number that makes the equation true in each of the } \\ \text { equations } 8 \times ?=48,5=[] \div 3,6 \times \text { ? }\end{array}$ |
| $\begin{array}{l}\text { Understand properties of } \\ \text { multiplication and the } \\ \text { relationship between } \\ \text { multiplication and division. }\end{array}$ | MAFS.3.OA.2.5 | $\begin{array}{l}\text { Apply properties of operations as strategies to } \\ \text { multiply and divide. Examples: If } 6 \times 4=24 \text { is known, } \\ \text { then } 4 \times 6=24 \text { is also known. (Commutative property } \\ \text { of multiplication.) } 3 \times 5 \times 2 \text { can be found by 3 } \times 5=15, \\ \text { then } 15 \times 2=30, \text { or by } 5 \times 2=10, \text { then3 } \times 10=30 .\end{array}$ |
| $($ (Associative property of multiplication.) Knowing |  |  |
| that $8 \times 5=40$ and $8 \times 2=16$ one can find $8 \times 7$ as |  |  |
| $8 \times(5+2)=(8 \times 5)+(8 \times 2)=40+16=56$. (Distributive |  |  |
| property.) |  |  |$\}$


|  |  | that 4 times a number is always even, and explain <br> why 4 times a number can be composed into two <br> equal addends. |
| :--- | :--- | :--- |
| Geometric measurement: <br> understand concepts of area <br> and relate area to <br> multiplication and to <br> addition. | MAFS.3.MD.3.7 | Relate area to the operations of multiplication and <br> addition. <br> a. Find the area of a rectangle with whole-number <br> side lengths by tiling it, and show that the area is the <br> same as would be found by multiplying the side <br> lengths. <br> b. Multiply side lengths to find areas of rectangles <br> with whole-number side lengths in the context of <br> solving real world and mathematical problems, and <br> represent whole-number products as rectangular areas <br> in mathematical reasoning. <br> c. Use tiling to show in a concrete case that the area <br> of a rectangle with whole-number side lengths a and b <br> + c is the sum of a x b and a x c. Use area models to <br> represent the distributive property in mathematical <br> reasoning. <br> d. Recognizing area as additive. Find areas of <br> rectilinear figures by decomposing them into non- <br> overlapping rectangles and adding the areas of the <br> non-overlapping parts, applying this technique to <br> solve real world problems. |
| Use the four operations <br> with whole numbers to |  | MAFS.4.OA.1.1 |
| solve problems. | Interpret a multiplication equation as a comparison, <br> e.g., interpret 35 = 5 7 as a statement that 35 is 5 <br> times as many as 7 and 7 times as many as 5. <br> Represent verbal statements of multiplicative <br> comparisons as multiplication equations. |  |
|  |  | Multiply or divide to solve word problems involving <br> multiplicative comparison, e.g., by using drawings <br> and equations with a symbol for the unknown number <br> to represent the problem, distinguishing <br> multiplicative comparison from additive comparison. |
| Gain familiarity with <br> factors and multiples. | MAFS.4.OA.2.4 | Solve multistep word problems posed with whole <br> numbers and having whole-number answers using the <br> four operations, including problems in which <br> remainders must be interpreted. Represent these <br> problems using equations with a letter standing for <br> the unknown quantity. Assess the reasonableness of <br> answers using mental computation and estimation <br> strategies including rounding. |
| Investigate factors and multiples. <br> a. Find all factor pairs for a whole number in the <br> range 1-100. <br> b. Recognize that a whole number is a multiple of <br> each of its factors. Determine whether a given whole <br> number in the range 1-100 is a multiple of a given <br> one-digit number. |  |  |



|  |  | does your answer lie? |
| :---: | :---: | :---: |
| Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. | MAFS.4.MD.1.2 | Use the four operations to solve word problems1 involving distances, intervals of time, and money, including problems involving simple fractions or decimals2. Represent fractional quantities of distance and intervals of time using linear models. (1See glossary Table 1 and Table 2) (2Computational fluency with fractions and decimals is not the goal for students at this grade level.) |
|  | MAFS.4.MD.1.3 | Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. |
| Perform operations with multi-digit whole numbers and with decimals to hundredths. | MAFS.5.NBT.2.5 | Fluently multiply multi-digit whole numbers using the standard algorithm. |
|  | MAFS.5.NBT.2.6 | Find whole-number quotients of whole-numbers with up to four-digit dividends and two-digit digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |
|  | MAFS.5.NBT.2.7 | Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. |
| Use equivalent fractions as a strategy to add and subtract fractions. | MAFS.5.NF.1.1 | Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2 / 3+5 / 4$ $=8 / 12+15 / 12=23 / 12$. (In general, $a / b+c / d=(a d+$ bc)/bd.) |
| Apply and extend previous understandings of multiplication and division to multiply and divide fractions. | MAFS.5.NF.2.3 | Interpret a fraction as division of the numerator by the denominator $(\mathrm{a} / \mathrm{b}=\mathrm{a} \div \mathrm{b})$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3 / 4$ as the result of dividing 3 by 4, noting that $3 / 4$ multiplied by 4 equals 3 , and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50 -pound sack of |


|  |  | rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? |
| :---: | :---: | :---: |
|  | MAFS.5.NF.2.4 | Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. <br> a. Interpret the product $(\mathrm{a} / \mathrm{b}) \times \mathrm{q}$ as a parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $\mathrm{a} \times \mathrm{q} \div \mathrm{b}$. For example, use a visual fraction model to show (2/3) $\times$ $4=8 / 3$, and create a story context for this equation. Do the same with $(2 / 3) \times(4 / 5)=8 / 15$. (In general, $(a / b) \times(c / d)=a c / b d$.) <br> b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. |
|  | MAFS.5.NF.2.5 | Interpret multiplication as scaling (resizing), by: <br> a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. <br> b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a / b=(n \times a) /(n \times b)$ to the effect of multiplying a/b by 1 . |
|  | MAFS.5.NF.2.6 | Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. |
|  | MAFS.5.NF.2.7 | Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. <br> a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1 / 3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1 / 3) \div 4=1 / 12$ because $(1 / 12) \times 4=$ 1/3. <br> b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div(1 / 5)$, and use a visual |


|  |  | laction model to show the quotient. Use the <br> relationship between multiplication and division to <br> explain that 4 $\div(1 / 5)=20$ because $20 \times(1 / 5) ~=~ 4 . ~$ <br> c. Solve real world problems involving division of <br> unit fractions by non-zero whole numbers and <br> division of whole numbers by unit fractions, e.g., by <br> using visual fraction models and equations to <br> represent the problem. For example, how much <br> chocolate will each person get if 3 people share $1 / 2$ lb <br> of chocolate equally? How many 1/3-cup servings are <br> in 2 cups of raisins? |
| :--- | :--- | :--- |
| Understand ratio concepts <br> and use ratio reasoning to <br> solve problems. | MAFS.6.RP.1.3 | Use ratio and rate reasoning to solve real-world and <br> mathematical problems, e.g., by reasoning about <br> tables of equivalent ratios, tape diagrams, double <br> number line diagrams, or equations. <br> a. Make tables of equivalent ratios relating quantities <br> with whole-number measurements, find missing <br> values in the tables, and plot the pairs of values on the <br> coordinate plane. Use tables to compare ratios. <br> b. Solve unit rate problems including those involving <br> unit pricing and constant speed. For example, if it <br> took 7 hours to mow 4 lawns, then at that rate, how <br> many lawns could be mowed in 35 hours? At what <br> rate were lawns being mowed? <br> c. Find a percent of a quantity as a rate per 100 (e.g., <br> 30\% of a quantity means 30/100 times the quantity); <br> solve problems involving finding the whole, given a <br> part and the percent. <br> d. Use ratio reasoning to convert measurement units; <br> manipulate and transform units appropriately when |
| multiplying or dividing quantities. |  |  |
| e. Understand the concept of Pi as the ratio of the |  |  |
| circumference of a circle to its diameter. |  |  |

$\left.\left.\begin{array}{|l|l|l|}\hline \begin{array}{l}\text { find common factors and } \\ \text { multiples. }\end{array} & & \\ \hline & \text { MAFS.6.NS.2.4 } & \begin{array}{l}\text { Find the greatest common factor of two whole } \\ \text { numbers less than or equal to 100 and the least } \\ \text { common multiple of two whole numbers less than or } \\ \text { equal to 12. Use the distributive property to express a } \\ \text { sum of two whole numbers 1-100 with a common } \\ \text { factor as a multiple of a sum of two whole numbers } \\ \text { with no common factor. For example, express 36 + 8 } \\ \text { as 4 (9 + 2). }\end{array} \\ \hline \begin{array}{l}\text { Apply and extend previous } \\ \text { understandings of } \\ \text { arithmetic to algebraic } \\ \text { expressions. }\end{array} & \text { MAFS.6.EE.1.1 } & \begin{array}{l}\text { Write and evaluate numerical expressions involving } \\ \text { whole-number exponents. }\end{array} \\ \hline & \text { MAFS.6.EE.1.2 } & \begin{array}{l}\text { Write, read, and evaluate expressions in which letters } \\ \text { stand for numbers. } \\ \text { a. Write expressions that record operations with } \\ \text { numbers and with letters standing for numbers. For } \\ \text { example, express the calculation "Subtract y from 5" } \\ \text { as 5 - y. } \\ \text { b. Identify parts of an expression using mathematical } \\ \text { terms (sum, term, product, factor, quotient, } \\ \text { coefficient); view one or more parts of an expression } \\ \text { as a single entity. For example, describe the } \\ \text { expression 2 (8 + 7) as a product of two factors; view } \\ \text { (8 + 7) as both a single entity and a sum of two } \\ \text { terms. } \\ \text { c. Evaluate expressions at specific values of their }\end{array} \\ \text { variables. Include expressions that arise from } \\ \text { formulas used in real-world problems. Perform } \\ \text { arithmetic operations, including those involving } \\ \text { whole-number exponents, in the conventional order } \\ \text { when there are no parentheses to specify a particular } \\ \text { order (Order of Operations). For example, use the } \\ \text { formulas } V=s^{3} \text { and } A=6 s^{2} \text { to find the volume and } \\ \text { surface area of a cube with sides of length } s=1 / 2 .\end{array} \right\rvert\, \begin{array}{l}\text { Understand solving an equation or inequality as a } \\ \text { process of answering a question: which values from a } \\ \text { specified set, if any, make the equation or inequality } \\ \text { true? Use substitution to determine whether a given } \\ \text { number in a specified set makes an equation or } \\ \text { inequality true. }\end{array}\right\}$

| independent variables. |  | thought of as the dependent variable, in terms of the <br> other quantity, thought of as the independent variable. <br> Analyze the relationship between the dependent and <br> independent variables using graphs and tables, and <br> relate these to the equation. For example, in a <br> problem involving motion at constant speed, list and <br> graph ordered pairs of distances and times, and write <br> the equation $=65$ to represent the relationship <br> between distance and time. |
| :--- | :--- | :--- |
| Solve real-world and <br> mathematical problems <br> involving area, surface <br> area, and volume. | MAFS.6.G.1.1 | Find the area of right triangles, other triangles, special <br> quadrilaterals, and polygons by composing into <br> rectangles or decomposing into triangles and other <br> shapes; apply these techniques in the context of <br> solving real-world and mathematical problems. |
|  | MAFS.6.G.1.2 | Find the volume of a right rectangular prism with <br> fractional edge lengths by packing it with unit cubes <br> of the appropriate unit fraction edge lengths, and <br> show that the volume is the same as would be found <br> by multiplying the edge lengths of the prism. Apply <br> the formulas V = l w hand V = b h to find volumes of <br> right rectangular prisms with fractional edge lengths <br> in the context of solving real-world and mathematical <br> problems. |



Language Arts Florida Standards (LAFS)


## Standards for Mathematical Practice

"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy)."

## Course Overview

This unit begins by introducing the concept of multiplication through literature and connecting it to addition, a math skill that students are familiar with. Continue to develop their understanding of the concept of what it means to multiply, how it is done, and why it is beneficial through a variety of concrete, visual, kinesthetic, tactile, and aural activities. Research shows that proficiency in learning multiplication facts is best attained through the use of a systematic approach which normally results in effortless recall. Therefore even the order in which the facts are taught is crucial. It is also imperative that students are able to count from 1 to 100 fluently and can skip count. Begin with these basic foundations. Then begin to teach the 0 through 12 multiplication facts, starting with those that are easiest for students to learn $-0,1,10,2,5$, and 9 . After that they will be ready for the more challenging facts of $4,7,3,8$, and 6 , respectively.

Explain respective rules for the 0 and 1 tables and the commutative property ( $3 \times 5$ $=5 \times 3,9 \times 7=7 \times 9$ ) after the 1 's. Use skip counting to introduce each table. Although skip counting works well, according to brain research, students respond well and retain information better when rhythm, rhymes, music, and/or movement are included in the learning process. For this reason, I created "Multiplication Jingles" and a method I call "Six, Sixes" to bring some rhyming and rhythm into the lessons.

In this unit there are my original multiplication rhythms and rhymes along with other resources because I know it takes a variety of learning strategies to reach the diverse learning styles of our students. Once the concept and all facts have been taught, it is important to provide practice, practice, and more practice. As you
continue to use these strategies, make them a springboard for creating your own methods. Also encourage your students to be creative and develop their own rhythms, rhymes, movements, and games.


## WHY USE RHYTHMS AND RYHMES WITH MULTIPLICATION?

## What do the experts say?

- Many tasks across all domains of mathematics and across many subject areas call upon the recall of basic multiplication facts as a lower-order component of the overall task. To enable students to focus on more sophisticated tasks such as problem solving, proficiency in basic facts and skills is an advantage (Ashcraft, Kirk, \& Hopko, 1998; Kilpatrick et al., 2001; Wu, 1999). Without procedural fluency and the ability to recall facts from memory, the student's focus during problem solving will be on basic skills rather than the task at hand, thus drawing attention away from the learning objectives of the task (Mercer \& Miller, 1992). If the student cannot perform these basic calculations without the need to use calculators or other aids, higher-order processing in problem solving will be impeded (Westwood, 2003). Monica Wong \& David Evans
- Music (songs) can be very effective in helping children to learn their multiplication tables. It works with the brain in manners that we are not yet fully aware of, but it is certain that music enhances learning and brain function...Using music and movement can be especially helpful for students who have math anxiety because it doesn't seem like "math" to them. By circumventing their 'mental block', they can learn the tables without difficulties-they see it as fun, and the 'math triggers' that the mind uses to 'block' the brain and produce the 'fear response' are never stimulated. Also,
younger siblings often learn the songs (and should be encouraged to learn) at the same time. Marcia L. Tate
- Consider this: there are some things that you know how to do that you will never forget how to do even if you live to be 100 (e.g., driving a car, riding a bike). You remember these activities because you were moving when you learned them which meant that the information was placed in one of the strongest memory systems in the brain: procedural or muscle memory. Procedural memory and episodic memory, another memory pathway, are both accessed when students are actively engaged in the learning process.


## Marcia L. Tate

## My personal experience?

I have been a Miami-Dade County public school teacher since the mid 1980's. During that time, I found that math is a subject that students need to be actively involved in at an early age so that they can develop the appropriate foundations for understanding and learning higher level mathematics. In the beginning I taught math from the textbook and it was strictly a pencil and paper activity. As time passed I noticed that some students needed a different method to connect with the concepts, remember the facts and procedures, and achieve success. I collaborated with a colleague to brainstorm ideas and he mentioned to me a strategy he used for multiplication which I eventually named Six Sixes. I thought it was a wonderful idea, tried it with my students, and saw an immediate change in their attitudes. I reflected on how to enhance this idea and decided that adding movement would be beneficial. My students thoroughly enjoyed this and began to create their own moves which led to increased fact mastery in their work. Since the Six Sixes was
such a huge success, I decided to create rhymes, add rhythm, and movement to accompany other multiplication facts as well. Those became my Multiplication Jingles. When I used both of these strategies in combination with other materials and ideas, an atmosphere of excitement and success was the norm in my mathematics classes.


On the following pages you will find the initial literaturebased lesson to jumpstart your multiplication unit. In addition to this I have included a variety of follow-up activities and suggestions that will help solidify the concept and develop fluency of the facts. Please feel free to contact me if you are in need of assistance or have questions. I would also like to hear your success stories.

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## Lesson Plan

## Overview:

Students will be introduced to the concept of multiplication through literature.

## Materials:

The Best of Times by Greg Tang
Manipulatives for counting (cubes, counters, etc.)
Handout with four examples of repeated addition
Multiplication as Repeated Addition worksheets

## Lesson:

1. Before the lesson, prepare the Smart Board or Promethean by drawing a square with this problem at the top $2+2+2=\ldots$. After defining multiplication for students, read The Best of Times by Greg Tang. (Depending on your students you can explore other books and choose the one that suits you best.) Once you have read the book, choose the number 2 to help them see the connection between adding and multiplying and explain that multiplying can be called repeated addition. You can call on groups of children to physically illustrate this by skip counting their eyes, ears, arms legs etc. Thoroughly developing their understanding of the concept is important. Depending on your students, you may need to spend more or less time developing their understanding of the concept of multiplication. Continue to relate multiplication to simple repeated addition with additional problems.

Eg. $2+2+2=6$ so $3 \times 2=6$
$5+5=10$ so $2 \times 5=10$

$$
3+3=6 \text { so } 2 \times 3=6
$$

$$
10+10+10+10=40 \text { so } 4 \times 10=40
$$

***Tell students the word "times" means the number of times a number is being added to itself so $4 \times 3$ means 3 added 4 times which equals 12. Provide similar examples.
2. I DO: The teacher will model how to show repeated addition and make the connection to multiplication. Students should be observing and taking notes as the teacher thinks aloud, demonstrating how to work through the process and provide rationale.
"The problem is $2+2+2=\ldots$. I need to add three 2 's to find out how many in all. There are three 2's so I need to draw three circles." On the Smart Board or Promethean draw 3 circles in the box.
"I need to put two items in each circle." Draw two items in the circles.
"This is repeated addition and I could add them three times or I could skip count." Skip count aloud and write the answer six.
"This shows that $2+2+2=6$ so $3 \times 2=6$. Under the objects write the multiplication equation."

Check for understanding.
3. WE DO: Give the students the handout with four examples of repeated addition in boxes and manipulatives. The teacher and students will work together to show how to find the answer for the first two repeated addition problems using manipulatives.

Call on a student to read the problem in box 1 . Guide students through the process, calling on volunteers to think aloud and demonstrate how to show the addition with these steps: students should read the problem, decide how many circles to draw, decide how many items to put in each circle, skip count to get the answer and explain that their box shows $5+5=10$ so $2 \times 5$ $=10$. Check for understanding. Repeat the WE DO with box 2 .
4. THEY DO: Have students follow the steps to complete box 3 with a partner, thinking aloud. Check for understanding by calling on a couple of partners to share how they completed the process. Repeat this step with box 4.
5. YOU DO: Have students turn the paper over. Write a repeated addition problem on the Smart Board or Promethean and instruct students to write it at the top of their paper. Tell them to that this time they will work alone to show what they know. Have them complete the problem using manipulatives and to be sure to write the multiplication equation at the bottom of the paper. Check for understanding as they work. Call on a couple of students to "show and tell" how they completed the process.
6. Assessment/Evaluation: Have students complete one of the Multiplication as Repeated Addition Worksheets.
7. Follow-up lessons: See additional activities section to continue working with the concept and building fluency. Keep in mind the importance of working in a sequential order as you work on learning the multiplication facts.


## FOLLOW-UP ACTIVITIES

## Materials in packet:

*Copy of Multiplication Notes
*Copy of Multiplication Jingles
*Copy of Six Sixes

## Materials to purchase:

*Literature related to multiplication
*Small index cards
*Rhythm instruments
*Incentives/rewards/awards

## Activities:

1. Once students have grasped the concept of multiplication, you can expose them to a variety of strategies to discover the answers to the equations and commit them to memory. Use the copy of "Multiplication Notes" to illustrate some of this variety to students. Since the goal for them is fluency, remind them that although there are many ways to arrive at the answer, it is best if they commit them to memory. You can illustrate the need for this if you show them a large number (three or four digits) and have them try repeated addition for that. At this point, tell them that multiplication becomes a "shortcut" for addition if they know the basic facts.

Eg. $435+435+435+435+435=2175$
That is the same as $5 \times 435=2175$
2. Once they understand the concept and are familiar with the various strategies, all that is left is practice, practice, practice. They need practice to gain fluency. Regular assessment is also important as it will encourage them to commit their facts to memory. Eventually they will master the facts. There are unlimited ways to provide practice. Incorporate skip counting in as many activities as possible. While standing in line you may have them skip count by 2, 3, 4, 5, etc. You can have them skip count as a warm up exercise or a bellringer. Once they know the count, insert the equations and have them fill in the numbers.
$2 \times 1=$ $\qquad$ , $2 \times 2=$ $\qquad$ , $2 \times 3=$ $\qquad$
$2 \times 4=\ldots$ and so on.
3. The Multiplication Jingles can be sung to an army cadence using 2 pats and 1 clap. Simple motions are added to the 0 and 1 tables such as making a large zero with arms overhead and using forefingers to show the 1 . Forearms are crossed to make the multiplication sign. When singing the 2's, 3 's 4's 5's and 10 's, have the children put out one finger each time they sing the number. 2(one finger), 4(two fingers), 6(three fingers), 8(four fingers).......keep the four fingers out while singing the rhyming phrase, and then continue...10(five fingers) etc.

On the multiplication chart, you will notice the highlighted 16 facts in the center, starting with $6 \times 6$ and ending with $6 \times 9$, then $7 \times 6$ etc. These 16 facts when boxed together have been known as the "Square of Despair" because students seemed to have the most difficult time remembering them. Using the Six Sixes, you can change that area into the "Box that Rocks"! When introducing the Six Sixes, write the left side of the list on the board and have students write all of those facts on one side on an index card the same way. When they finish, have them turn the card over and write the right side of the list on the other side of the index card the same way. Tell them that on the left side, they will be saying those ten facts exactly how they see it saying 6 times 6 equals 36 etc. Then when they flip their cards over, they will be leaving out the word "times" and "equal" saying " 6,6 , 36 ". Notice that the Six Sixes only includes 10 facts from the "Square of Despair" due to the commutative property.

Practice with them daily for several days. After a few days of practice, Add two pats and two claps to the ten facts that eliminate saying times and equals. Pat, pat, clap, clap has the same rhythm as saying " $6,6,36$ " all the way through to " $9,9,81$ ".

As they become more fluent you may wish to add motions using the same rhythm. Children enjoy healthy competitions and challenges. As an extra challenge, ask the children to say their Six Sixes in 12 seconds or less and have them try to beat the timer.
4. As an extension, students can write and illustrate their own problems to be solved by others and can create their own multiplication story books.
5. Finally, be sure to assess your students' progress regularly. Give quizzes on the facts weekly. Use the same facts over and over again until you see a desired mastery. Then you may change facts. Be sure to vary the method of assessment (orally, with paper and pencil, and on the computer, etc.).

## Make it Stick! - Ideas for Practicing Facts

There are many ways students can practice their facts besides rote memorization. Try some of the following ways.

* Factor Ping-Pong - Students rally back and forth saying the next consecutive multiple of a factor.
* Playing Cards - Students deal two cards and multiply together to win points.
* Grouping Manipulatives - M \& M’s, pencils, cheerios, etc.



## Resources

## HANDOUTS:

*Copy of multiplication chart - filled *Copy of multiplication chart - blank

## WEBSITES:

www.aceeducational.com/ (Multiplication CD's, DVD's, and rhythm sticks)
http://www.aplusmath.com/cgi-bin/Flashcards/Custom_Flashcards (make flashcards)
http://www.aplusmath.com/Games/Concentration/Multiplication_Concentration.ht ml (multiplication game of concentration)
http://www.mathsisfun.com/timestable.html (online multiplication quizzes)
http://www.multiplication.com/ (multiplication resources)
http://www.playkidsgames.com/games/mathfact/mathFact.htm (online math fact practice)
http://www.prometheanplanet.com/en-us/ (Register and begin to identify/create flipcharts to accompany and enhance your lessons.)
https://www.reflexmath.com/ (activities for math fluency)
www.teachertoolsinc.com/ (Multiplication CD's, DVD's, and rhythm sticks)
www.themathparty.com (music to learn math concepts)
http://www.vrml.k12.la.us/curriculum/schoolhouserock/math_shr.htm (Schoolhouse Rock videos)


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## Multiplication Notes

What is multiplication?

1. Multiplication is an operation that finds the total number of items that are in equal groups. A product is the answer in a multiplication problem.
2. Multiplication is repeated addition. This means the same number is added over and over again. It can be called a short cut for adding ():

## EQUAL GROUPS



Six circles with four dots. 6 circles with 4 dots. $6 \times 4=24$ dots

Repeated Addition

$$
4+4+4+4+4+4=24
$$

4 is added over and over and over -- 6 times - 6X

You can show multiplication with an array which is an ordered arrangement of dots in equal rows and columns.
$6 \times 4=24$ dots

You can skip count to find a product.


Skip count by 4 six times.
$6 \times 4=24$

You can use a multiplication table to find answers to multiplication problems. It takes time to create your own table. You may want to use a ready-made table if that is acceptable.

Go down

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 0 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 |

## Go across

Find the answer where the column and row meet.

You can use a calculator to find answers and to check your work as well. A calculator may take some time too.


The 9 times tables have special tricks.

1. The Ladder - Write the numbers 0-9 going down. The write the numbers $0-9$ going up right next to the first set of numbers. The paired numbers are the answers to the 9 times tables.

$$
\begin{gathered}
\text { going } \begin{array}{c}
09 \rightarrow=9 \times 1 \\
\text { down } \\
18 \rightarrow=9 \times 2 \\
27 \rightarrow=9 \times 3 \\
36 \\
45 \\
54 \\
63
\end{array} \\
9 \times 8=\longleftarrow 72 \\
9 \times 9=<81 \\
9 \times 10=\longleftarrow 90
\end{gathered}
$$

2. You can also use Chisenbop which is a Chinese counting method of multiplying. Try it with the 9 times tables.


ones $\rightarrow$

For $4 \times 9$ bend finger number 4 down. The number of fingers to the left of the bent finger are tens and the number to the right are ones so $4 \times 9=36$ !

You can always ask, and ask, and ask and ask, and ask, $\qquad$ for answers forever.

What is $6 \times 4$ ?
What is $6 \times 4$ ?

What is $6 \times 4$ ?
What is $6 \times 4$ ?

Or, you can learn them and memorize them and keep them in your brain forever! How?

Write them, sing them, say them, rap them, test yourself, use flashcards and study as much as you need to until you know them well.

## Properties of Multiplication - Reduces fact studying by $1 / 2$ ())!

1. Commutative Property

$$
6 \times 4=4 \times 6
$$

$$
24=24
$$

2. Identity Property of Multiplication

$$
\square \times 1=\square
$$

## 3. Zero property of Multiplication

$$
\square \times 0=\square
$$

$\qquad$

## Multiplication Jingles

0 - All I know about zero is, the answer never changes. $0 \times(\square)=0$

1- When you multiply by 1 , the answer's always the other one. $1 \times(\square)=(\square)$

2-2,4,6,8, these times tables are really great!, 10, 12, 14,16 , I'll be done when I get to 18.

3-3, 6,9 , and 12, we can learn it by ourselves, 15,18 , 21, just two more and I'll be done, 24 and 27, I can brag to my friend Kevin.

4-4, 8, 12, 16, do you have to be so mean? 20, 24, 28, these times tables are really great, 32 and 36 I'll say the fives just for kicks.
$5-5,10,15,20$, we are really learning plenty, 25,30 , 35,40 , don't you dare call me shorty, 45 is at the end. Why don't we do them again?
$10-10,20,30,40$, did I hear you call me shorty? 50, 60 , 70, 80, have you ever been to Haiti? 90, and 100 too, times tables are fun to do!

## Six Sixes

$6 \times 6=36$
6.636
$6 \times 7=42$
6.742
$6 \times 8=48$
6.848
$6 \times 9=54$
6.954
$7 \times 7=49$
7.749
$7 \times 8=56$
7.856
$7 \times 9=63$
7.963
$8 \times 8=64$
8.864
$8 \times 9=72$
8.972
$9 \times 9=81$
9.981
$\qquad$

Topic: Multiplication as Repeated Addition - Worksheet 1
How many items are present?

1.


Three times of __ cows = ___ cows


3 fish $\quad 3$ fish $\quad 3$ fish $\quad 3$ fish
2.

+ __-__ + $\qquad$

$\qquad$
$\qquad$ $=$ $\qquad$

Four times of $\qquad$ fish $=$ $\qquad$ fish


5 elephants


5 elephants
3.


Two times of $\qquad$ elephants = $\qquad$ elephants

2 ducks 2 ducks 2 ducks 2 ducks 2 ducks 2 ducks 2 ducks
4.


Seven times of __ ducks = $\qquad$ ducks

5.


Four times of $\qquad$ bags $=$ $\qquad$ bags
$\qquad$

Topic: Multiplication as Repeated Addition - Worksheet 2
How many items are present?

1.


Two times of $\qquad$ chairs $=$ $\qquad$ chairs


2 buses

$\square$


Four times of __ buses = $\qquad$ buses
3.


Three times of $\qquad$ pens $=$ $\qquad$ pens
4.

$$
z_{--}+_{---}^{+}+_{---}+_{---}+_{---}=_{----} X_{----}=
$$

Five times of $\qquad$ huts = $\qquad$ huts

5.


Three times of __ combs = ___ combs
$\qquad$

Topic: Multiplication as Repeated Addition - Worksheet 3
How many items are present?
2 cars 2 cars 2 cars 2 cars
1.


Four times of __ cars = ___ cars


4 watches
4 watches
4 watches


Three times of __ watches = $\qquad$ watches


6 butterflies

## 6 butterflies

3. 
```
___-_ + _____- = _____- X ___-_ = ___-_
```

Two times of butterflies = $\qquad$ butterflies
$\qquad$

4.


Four times of __ dogs = $\qquad$ dogs


2 balls 2 balls 2 balls 2 balls 2 balls
5.


Five times of $\qquad$ balls = $\qquad$ balls
$\qquad$

Topic: Multiplication as Repeated Addition - Worksheet 4
How many items are present?


3 rats


3 rats


3 rats


3 rats
1.


Four times of __ rats = $\qquad$ rats

2.


Three times of __ cats = $\qquad$ cats


4 hats
4 hats
4 hats
3.


Three times of $\qquad$ hats $=$ $\qquad$ hats

4.
2 birds 2 birds
2 birds 2 birds 2 birds 2 birds 2 birds


Seven times of __ birds = $\qquad$ birds

5. 3 bells


3 bells
$\square$


$$
=
$$

$\square$ X $\qquad$ $=$ $\qquad$

Three times of __ bells = $\qquad$ bells
$\qquad$

Topic: Multiplication as Repeated Addition - Worksheet 5
How many items are present?


5 lamps


5 lamps

```
___-_ + ______ = ______ X______ = ____-_
```

Two times of __ lamps = $\qquad$ lamps


3 pencils 3 pencils 3 pencils $\quad 3$ pencils 3 pencils 3 pencils
2.
_-_-_ $+_{\text {_-_-_ }}+_{\text {_-_-_ }}+_{\text {_-_-_ }}+_{\text {_-_-_ }}^{+}$
$=\ldots---X_{\text {_---- }}=$

Six times of __ pencils $=$ $\qquad$ pencils


2 keys


2 keys
3.


Two times of $\qquad$ keys = $\qquad$ keys
$\qquad$

4. 6 fish
$\qquad$ $+\quad$ _-_ $\qquad$ $+\quad=$ $\qquad$ $X_{\text {_-_- }}=$ $\qquad$

Three times of $\qquad$ fish $=$ $\qquad$ fish

5.

$$
4 \text { donkeys }
$$

4 donkeys


Two times of $\qquad$ donkeys = $\qquad$ donkeys

Student Name: Date:

## Multiplication Chart 0-12

| $\mathbf{x}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 |  |  |  |  |  |  |  |  |  |  |  |  |  |

Student Name: $\qquad$

Answers

| $\mathbf{X}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathbf{1}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $\mathbf{2}$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| $\mathbf{3}$ | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| $\mathbf{4}$ | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| $\mathbf{5}$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| $\mathbf{6}$ | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| $\mathbf{7}$ | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| $\mathbf{8}$ | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| $\mathbf{9}$ | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| $\mathbf{1 0}$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| $\mathbf{1 1}$ | 0 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| $\mathbf{1 2}$ | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

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