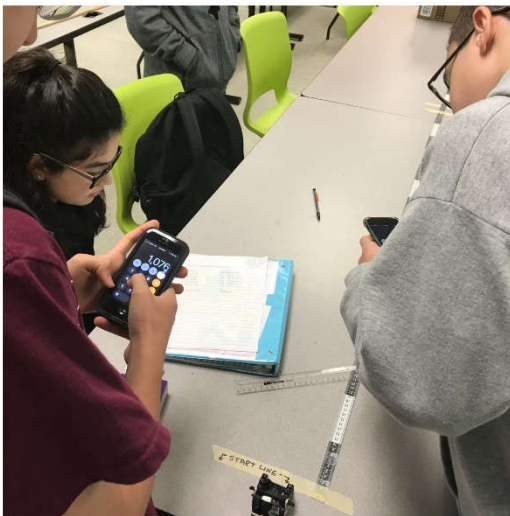
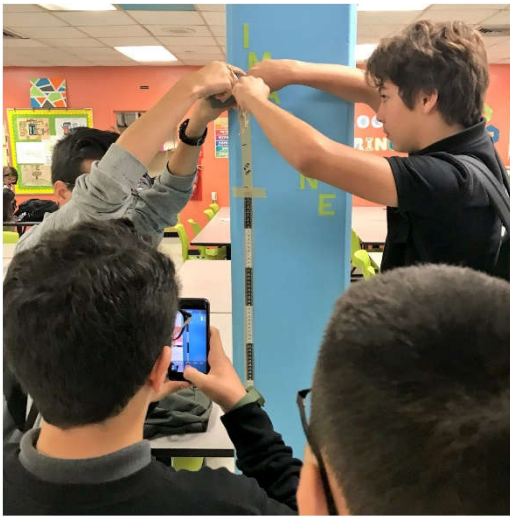


PROJECT-BASED ALGEBRA

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PROJECT GOALS AND TARGET AUDIENCE

The purpose of this project is to offer students authentic learning opportunities in mathematics. Each lesson aims to highlight mathematics in STEM applications by having students participate in project-based learning opportunities. At the end of each activity, students are asked to summarize their findings, allowing them to practice content specific academic writing skills. Overall, this project aims to reduce the barriers that exist between subjects in school and real-world applications, offering students an engaging and exciting way to apply mathematics to the world around them.

This curriculum packet was explicitly developed for use with Algebra 1 students in grades 7-9. These activities tend to be low floor high ceiling in nature and can work well in a differentiated classroom with modification. Additionally, these activities can be (and have been) modified for use with students from grades 4 through 12.

Teachers are invited to complete a sample activity and work through the process of modifying activities to meet a diverse set of learners by attending a workshop at the Idea Expo with Dale Adamson.

STANDARDS ALIGNMENT

***Please note that while these projects were created for Algebra 1 students, and the standards below reflect that fact, these projects have been used with students from grades 4 through 12 with modification to meet grade-specific standards.**

FLORIDA STANDARDS- MAFS HIGH SCHOOL STANDARDS

MAFS.912.F-IF.2.6

Domain-Subdomain: Functions: Interpreting Functions

Cluster: Interpret functions that arise in applications in terms of the context.

Description: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

MAFS.912.S-ID.2.6

Domain-Subdomain: Statistics & Probability: Interpreting Categorical & Quantitative Data

Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

Description: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.

Emphasize linear, and exponential models. Informally assess the fit of a function by plotting and analyzing residuals. Fit a linear function for a scatter plot that suggests a linear association.

MAFS.912.S-ID.3.7

Domain-Subdomain: Statistics & Probability: Interpreting Categorical & Quantitative Data

Cluster: Interpret linear models.

Description: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

MAFS.912.A-SSE.2.3

Domain-Subdomain: Algebra: Seeing Structure in Expressions

Cluster: Write expressions in equivalent forms to solve problems.

Description: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. Use the properties of exponents to transform expressions for exponential functions. For example the expression can be rewritten as \approx to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

CCSS- ENGLISH LANGUAGE ARTS

CCSS.ELA-LITERACY.RST.8.3

Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.

CCSS.ELA-LITERACY.RST.8.4

Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 6-8 texts and topics.

CCSS.ELA-LITERACY.RST.8.7

Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

CCSS.ELA-LITERACY.WHST.8.7

Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.

CCSS.ELA-LITERACY.WHST.8.4

Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

PROJECT 1: LINEAR BUNGEE JUMPING (2-3 days)

OBJECTIVE

The goal of this project is to use bungee jumping to introduce students to scatterplots and to reinforce concepts of linear functions such as slope and y-intercept. Through this investigation, students will collect and plot data, create a line of best fit, and estimate the equation of their line of best fit. Students will then use that line of best fit to extrapolate the amount of bungee cord that would be needed to bungee from the top of the building. Finally, students will have the opportunity to see how well their data collection and line of best fit worked by watching their completed bungee in action.

MATERIALS

- Size #32 rubber bands
- Measuring tapes
- Small brass weights (or other dense objects that can be dropped)
- Unsharpened #2 pencils
- Optional: Student devices (for slow-motion video)
- Optional: Classroom devices (for using technology to confirm a line of best fit)

THE HOOK

- Start the period with a brief clip of bungee jumping: <https://youtu.be/zG22gQydPVQ?t=47s>
- Have a brief discussion with students about why math might be necessary in bungee jumping.
 - Key Point 1: You don't want to have too much bungee (in length or flexibility), or a jumper could hit the ground and get severely injured.
 - Key Point 2: People weigh different amounts and may need different strength/length bungee cords.
 - Key Point 3: You don't want to give too little cord, because that wouldn't be exciting for clients.
- How can we find the sweet spot? How can we get as close to the ground as possible while ensuring safety?
- Today's activity will collect data and try to answer that exact question.

SETUP (BEFORE STUDENTS ENTER THE CLASSROOM)

- Plan on having small groups of 2-3 students and prepare stations for each group. Each station needs about 15 rubber bands, a pencil, measuring tapes, and a brass weight (or other reasonably dense object).
- Adhere measuring tapes to the wall vertically. If the tapes are short, you may need to stick multiple tapes together. It is recommended that you have at least 2 meters of measuring tape on the wall for each group.
- As a culminating activity, students will test their bungee cords from a pre-determined height. After deciding where students will try their final bungee cords, you will need to measure the drop distance. Possible drop locations include the second or third floor of a building or from the roof if you have access. The best way to measure that distance is to drop a weighted line from the top and subsequently measure the length of the line.

PROCEDURE

- Divide students into groups of 2-3 students.
- Students will be creating their own bungees cords using rubber bands and testing the “drop distance” of different length bungees.
- Each group needs to decide on a standardized procedure for connecting rubber bands, and then continue to use that same procedure for the duration of the project. (The instructor may want to explore this ahead of time and offer a plan to students).
- Groups will first connect a single rubber band to the brass weight. We will call this band the “harness,” and it will not count towards the bungee’s rubber band count.
 - Using different size brass weights will ensure that different groups are not sharing data and answers along the way.
- Next students will add one rubber band at a time and measure how far the weight falls. To measure how far the weight drops, students will place the eraser side of an unsharpened number 2 pencil on the wall at the top of the measuring tapes. Then they will slide the rubber band onto the pencil. Next, one student will drop the weight, and a teammate will approximate the lowest point the weight falls to on the measuring tape.
- Once each group has a general idea of the distance the weight will fall, groups should do a second drop, attempting to take a more exact measurement. (Optional: Most smartphones have access to a slow-motion feature that works great for increasing the precision of measurements in this step. If you are comfortable doing so, I would recommend allowing students to use one phone per group in this step).

- Students will repeat the above procedure multiple times, adding more rubber bands each time. They should record all data in the table provided on the student handout.
- After collecting all their data, students should create a scatterplot of the data and approximate a line of best fit (Remind students that a line of best fit does not have to pass through the origin)
- Using their line of best fit, students can now estimate the number of rubber bands needed to drop their weight from the predetermined drop location.
 - Should students round up or down on the number of rubber bands needed?
- Have students build their final bungee and attach the weight/harness.
 - Make sure that groups use the same weight for all measurements and the final drop to account for any differences in the brass weights used)
- Finally, the students (or the teacher if using the roof of the building) will drop the weights and allow students to see how well they were able to model bungee cord drop distance using linear functions

FOLLOW-UP/STUDENT WORK PRODUCT

1) Students should have completed the worksheet, including all questions not specified in the directions attached.

2) Optional: Students can write a post-lab summary that outlines the purpose of the activity, the data collection process, and all results/conclusions. This gives students an opportunity to reflect on the purpose of the activity formally and to practice writing academically across the curriculum.

STUDENT HANDOUT

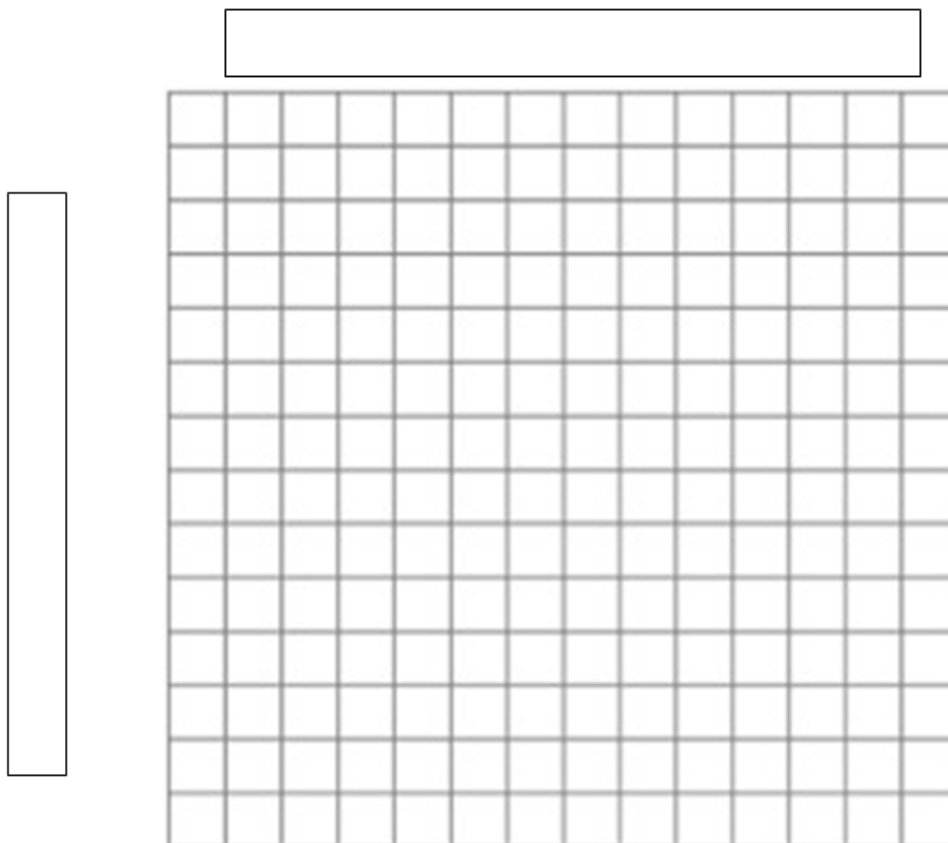
Name: _____ Partners: _____ Date: _____

Bungee Jumping with Algebra

What is the investigation about?

Number of rubber bands							
Distance fallen							

Use the above data to create a scatter plot on the grid below. Label the graph/axes and write your own scale (The x-axis and y-axis do not have to have the same scale). Finally, estimate the line of best fit.



Once your scatter plot is complete:

Use the metal rod to create a line of best fit. Find two of your points that are closest to the line of best fit. Use those two points to generate an equation in slope-intercept form. Show your work here.

Point: _____ Point: _____ Rate of Change: _____

What is the equation for the line of best fit? _____

What does the slope represent in this equation?

What does the y-intercept represent in this equation?

Make a prediction: Based on your line of best fit, how many rubber bands should be used to safely drop your weight from a height of _____?

Answer: _____ rubber bands

PROJECT 2: PAPER ROCKETS AND QUADRATIC MODELS

OBJECTIVE

The purpose of this project is to model the flight of a rocket using quadratic functions. In this investigation, students will reinforce vocabulary and concepts such as vertex, axis of symmetry, and zeros. Using a quadratic model, students will be able to approximate the maximum altitude achieved by their rocket. Because we are neglecting air resistance, this calculation serves only as an approximation of the actual height attained by the rocket.

MATERIALS

- Paper Rocket Launcher (<https://www.rocketblasters.com/shop/>)
- Air compressor
- Extension cord
- Printer Paper
- File folders
- Scotch Tape
- Stopwatches
- $\frac{1}{2}$ " PVC pipe (approximately 18-inch piece per group)
- Optional: Foam, hot glue, duct tape, etc.

THE HOOK

Teacher-led discussion with students centered on parabolic motion:

- What do you know about parabolic motion?
- Where have you seen that kind of motion before?
- How can we estimate how high an object is flying when in flight? (The objective of today's activity)

PROCEDURE

ROCKET CONSTRUCTION

- Break students into groups of 2 for rocket construction
- Build the rocket body:
 1. Give each group a piece of $\frac{1}{2}$ " PVC pipe
 2. Wrap a single sheet of printer paper tightly around the PVC pipe and use scotch tape to seal the seam.

3. Slide the rocket body on and off the PVC to make sure that it can come on and off fairly easily (although a little friction between the body and the PVC pipe is ideal).
- Create and attach fins:
 1. Have students select a shape that they want to use for fins (triangles, quadrilaterals, and semi-circles all tend to work).
 2. Cut the desired number of fins out of the **file folders** to make sure they are stable enough to create drag on the back end. Fins made out of regular paper will not be rigid enough to stabilize the rocket.
 3. Evenly space the fins and use tape to attach them to the body of the rocket.
 - Create a nose cone:
 1. Seal the nose, by taping it closed, pinching it closed, or creating a nose cone out of foam/cardboard.
 2. Make sure that the cone is well secured. Nose cones have a tendency to blow off if they are not well attached.
 - For additional information about rocket construction, consult the NASA Rocketry guides found in the resources section of this guide.

ROCKET LAUNCH

- Launch student rockets, having students record hangtime. Record and average the times from three different stopwatches.
- If time permits, you can launch each rocket multiple times (provided that the rockets survive the first flight).

MODELING WITH A QUADRATIC FUNCTION

1. Use the roots of the function ($t=0$ and $t=\text{hangtime}$) to write a quadratic function in factored form. For example, with a hang time of 6 seconds the equation would be:

$$h = (t - 0)(t - 6)$$
2. Add a coefficient of 4.9 to account for gravity, giving an equation of:

$$h = 4.9(t - 0)(t - 6)$$
3. Multiply out the function to convert it to standard form:

$$h = 4.9t^2 - 29.4t$$
4. Use the equation to determine the height of the rocket at the vertex. Students should be challenged to figure out what time to use in this calculation (Half of the flight time because of the symmetry of parabolas).
5. Sketch and label the resulting parabola on the worksheet.

Name: _____ Partner: _____ Date: _____

Paper Rockets: Modeling with Quadratic Functions

Objective: What is the purpose of this activity?

Rocket Design: In the boxes below, sketch the key features of your rocket. Make sure to measure and label all components, including what material was used to make each part.

Full Rocket Design

Fin Design

Nosecone Design

Data- Rocket Hangtime: In the chart below record the three stopwatch times from your first launch and calculate the mean. If you are permitted additional launches, you may need to use the additional spaces and to calculate an overall mean hangtime.

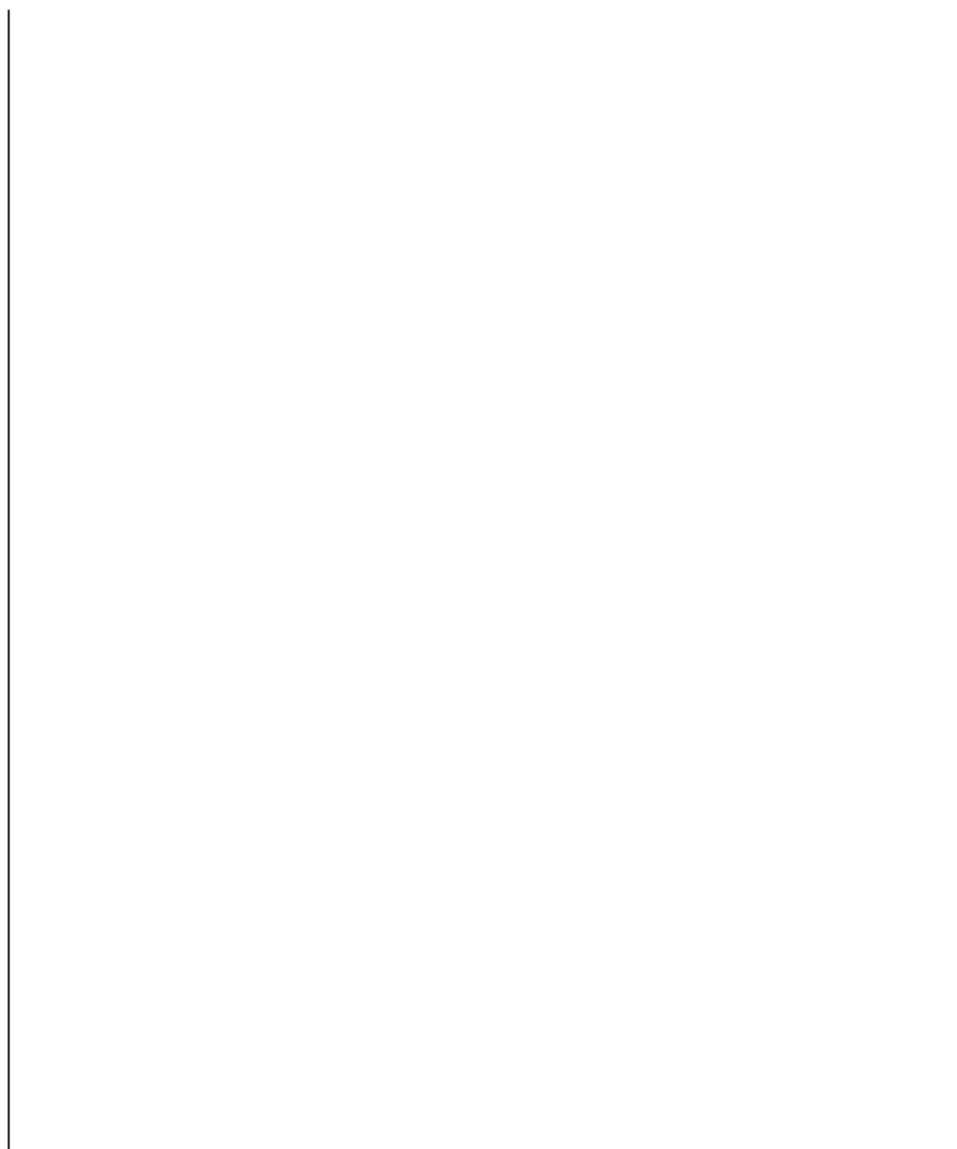
Trial	Watch 1	Watch 2	Watch 3	Mean Hangtime
1				
2				
3				
Mean				

Calculations:

1. What are the roots of your function?
2. Use your roots to write a quadratic function in factored form.
3. In front of your function, add a coefficient of $4.9 \text{ (m/s}^2\text{)}$ to account for the acceleration due to gravity.
4. Simplify your function and rewrite in standard form.
5. After how much time did your rocket reach the vertex?
6. Use the amount of time from Step 5 and the function you came up with in Step 4 to calculate the height of your rocket at its vertex.

7. The number you calculated in Step 6 is expressed in meters. Convert to feet ($1 \text{ m} = 3.28 \text{ ft}$)

8. Use the roots, vertex, and axis of symmetry to sketch the flight of your rocket. Make sure to title your graph and label each axis. The x-axis and y-axis do not have to have the same scale.



PROJECT 3A: CATAPULTS IN STATISTICS

PROJECT GOAL

The project goal is to engage students in meaningful and authentic collection and analysis of data. Given that Algebra 1 standards now include statistics, it is imperative that we provide students opportunities to collect, process, and interpret data. Rather than starting with an arbitrary data set from a textbook, this project proposes that students can collect authentic data from STEM applications to practice using statistics. There are many variations of this project, but this project assists students in understanding the significance of mean and standard deviation as measures of center and spread.

MATERIALS

- VEX Catapults, 3D printed catapults
- Rubber bands
- Popsicle sticks
- Recycled materials (student provided)
- Miscellaneous supplies (scissors, glue, tape, etc.)
- Measuring tapes
- Calculators
- Optional: Student smartphones

THE HOOK

- Set the scene:
 - Each group is a small village seeking to expand their reach in the world through the conquest of neighboring villages.
 - Each group of students will design and construct a catapult, with the goal of conquering other villages. Upon completing their catapult, groups need to figure out just how far their projectile will travel and how reliable their catapult is.
 - Each group will use their data to challenge other groups until one group emerges victorious.

SETUP (BEFORE STUDENTS ENTER THE CLASSROOM)

- Arrange workstations in a uniform grid pattern (3 x 3 works best)

PROCEDURE

- Briefly explain the idea of a catapult (or ballista) for any students unfamiliar with the mechanics. Each group is charged with constructing their very own catapult from recycled materials, rubber bands, and other everyday objects.
- Create groups of 3 students and let them find a place in the room to begin construction.
- Allow students 2-3 days to brainstorm and construct their catapults. During the design process, students are encouraged to prototype and test their catapult as much as possible.
- Each group will need to decide whether it is more important for them to make a catapult that travels far or one that is reliable (Distance will be calculated using mean launch distance. Reliability will be calculated using standard deviation).
- Once students have a final design, they must collect ten trials worth of data and formally calculate mean and standard deviation. Measurements can be taken using measuring tapes and the slow-motion feature of a smartphone.
- Playing the game:
 - Each group will set up their “village” in the same space that they chose to work on their design. Their village will be represented by a circular target that is 1 foot in radius and has three concentric rings.
 - The outermost ring is worth 1 point, the second ring 2 points, and the innermost ring is 3 points.
 - Using a random number generator (www.random.org), select a group. That group will be permitted to “attack” another group’s village. The two villages will alternate shots until one group has accumulated 3 points (If the catapults are well made this number could be increased to 5 or 7 points).
 - Once a village reaches 3 points, it takes over the village and now has access to two catapults, two villages, and two random numbers.
 - Select a new random number and repeat this process until one village is left standing. If a group is selected or attacked and it owns more than one village it can launch counter attacks using any ONE of its catapults and ONE of its village sites. If defeated, a group with multiple villages forfeits all its villages to the winning team.

EXTENSION

- If available, students can compare their data to data generated by VEX or 3D printed catapults. Students should note the significance of any difference in mean and standard deviation.

STUDENT HANDOUT

Groups should complete one of these forms for each prototype. They must complete at least two prototypes over the course of the building period.

Prototype Summary

Complete one form per prototype

Sketch

Data/Calculations:

Data (x)	Mean (\bar{x})	$d = x - \bar{x}$	d^2	Standard Deviation= $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$
TOTAL Σd^2				Mean = S.D. =

Conclusion: Based on your data, do you think this catapult will work well for this game? Why or why not?

ADDITIONAL RESOURCES

NASA Rocket Activity:

https://www.nasa.gov/pdf/295786main_Rockets_Adv_High_Power_Paper.pdf

Paper Rockets:

https://er.jsc.nasa.gov/seh/Paper_Rockets.pdf

Random number generator:

www.random.org

Research Spotlight on Project-Based Learning:

<http://www.nea.org/tools/16963.htm>

The Power of Project Learning:

<http://www.scholastic.com/browse/article.jsp?id=3751748>

Purchasing Rocket Launchers:

<https://www.rocketblasters.com/shop/>

GeoGebra (Useful tool for quickly running statistics):

<https://www.geogebra.org/download>

Desmos (Online graphing calculator):

<https://www.desmos.com/calculator>

VEX Catapults:

<https://www.hexbug.com/vex-robotics-catapult.html>

3D Printable Catapults (If you have access to a 3D printer):

<https://www.thingiverse.com/search?q=catapult&dwh=745b7222c229c3c>